A PROPERTY OF A SIMPLY ORDERED SET[1]

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Sierpinski[2] has shown that the set of real numbers of the interval (0, 1) may be decomposed into c disjoint subsets, each of power less than c, and such that the sum of every c of these subsets has at least one point in common with every perfect subset of the interval.

The object of the present paper is to show that the same method of proof may be used to prove an analogous theorem concerning a more general type of set.

DEFINITIONS. *A simply ordered set M is a set such that if any two of its elements are given it is known which one precedes.*

A subset of M is said to be cofinal (coinitial) with M if no element of M follows (precedes) all the elements of the subset.

An η_α subset of M is one which is neither cofinal nor coinitial with any subset of M of power less than $\aleph_\alpha$ and which contains no pair of neighboring subsets both of which have power less than $\aleph_\alpha$.

Various properties of simply ordered sets M containing everywhere dense η_α subsets, including a discussion of the perfect subsets of M, were discussed by the writer in a previous paper.[3]

THEOREM 1. *Let M be a simply ordered set containing an everywhere dense η_α subset N. There exists a decomposition of M into $2^{\aleph_\alpha}$ disjoint subsets, each of power less than $2^{\aleph_\alpha}$, and such that the sum of every $2^{\aleph_\alpha}$ of these subsets has at least one point in common with every perfect subset of M.*

PROOF. Let ϕ be the smallest ordinal number of power $2^{\aleph_\alpha}$. A transfinite sequence of type ϕ formed of all the points of M exists, namely,

$$m_1, m_2, m_3, \cdots, m_\xi, \cdots, \qquad \xi < \phi. \tag{1}$$

The perfect subsets of M having power $2^{\aleph_\alpha}$ may be arranged in the form of a transfinite sequence of type ϕ as follows:

$$M_1, M_2, M_3, \cdots, M_\xi, \cdots, \qquad \xi < \phi. \tag{2}$$

Let us now define a transfinite sequence $\{H_\xi\}_{\xi<\phi}$ of subsets of M: H_1 is formed of the single element m_1.

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[1] Presented to the Society, November 22, 1940.

[2] Fundamenta Mathematicae, vol. 24 (1935), pp. 8–11.

[3] Proceedings of the Royal Society of Canada, Section III, vol. 22 (1928), pp. 225–239.