IMBEDDING THEOREMS IN DIFFERENTIAL GEOMETRY*

T. Y. THOMAS

The matters which I shall discuss today will be largely concerned with the general problem of imbedding a coordinate manifold of class C^r in a euclidean space of sufficiently many dimensions. I imagine you all have a fair idea as to what is meant by an n-dimensional manifold of class C^r. Briefly this may be described as a Hausdorff space each point of which admits a neighborhood homeomorphic to the interior of a sphere in an n-dimensional euclidean space. We then suppose that the coordinate systems which may be introduced into these neighborhoods by these homeomorphisms are such that the coordinate relations which exist in the intersection of two such coordinate neighborhoods are of class C^r, that is, possess continuous partial derivatives to the order r inclusive. Of course when we consider imbedding theorems in differential geometry, which is the title of this address, one usually thinks, possibly from historical reasons incidental to the development of the subject, that the given space is endowed with a Riemann metric, and is then concerned with the problem of isometric imbedding. We shall also have something to say about the problem of isometric imbedding, although in doing so we shall limit ourselves to results of a general character. Of necessity most of the mathematical details must be omitted from our discussion however interesting these may be, but here and there certain detailed considerations will be introduced when it appears that these are directly understandable and may be treated with dispatch.

Before proceeding to the discussion of our particular subject I should like to say a few words about the analogous purely topological imbedding problem. I have in mind principally the classical result of Menger [1] and Nöbeling [2] to the effect that every n-dimensional compact metric space is homeomorphic to a subset of the euclidean space of $2n+1$ dimensions. Let A and B be two compact metric spaces and let M denote the set of all continuous maps of A into subsets of B. If f and f' are two elements or points of M, we define the distance between these points to be the maximum value of the distance of the points $f(x)$ and $f'(x)$ as x runs over the points of the space A. With this definition of distance M becomes a metric space and may readily be shown to be complete, that is, every Cauchy sequence in M con-

* An address delivered before the Stanford meeting of the Society on April 15, 1939, by invitation of the Program Committee.