

CONCERNING CERTAIN LINEAR ABSTRACT SPACES AND SIMPLE CONTINUOUS CURVES*

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The first section of this paper is given over mainly to the investigation of linear Hausdorff spaces. However, the principal object of the paper is to characterize topologically that class of point sets used in the geometry for lines, namely, the class of simple continuous curves. This has already been done by R. L. Moore,[†] by R. L. Wilder,[‡] and by myself.[§] The results of this paper generalize the results just referred to mainly by omitting all compactness requirements. As a matter of fact, it will be shown that any nondegenerate linear continuum lying in a Moore space is a simple continuous curve, and that any nondegenerate linear connected subset of a Moore space is homeomorphic with a simple continuous curve.

1. Certain results for Hausdorff spaces.

DEFINITION. *A space is said to be strongly regular at a point P provided that, if R is a region containing P , then there exists in R a domain D containing P whose boundary is a subset of the sum of a finite number of continua lying in $R - D$.|| A space is said to be strongly regular provided that it is strongly regular at every one of its points.*

THEOREM 1. *If P is a point of a connected Hausdorff space M and M is strongly regular at P , then M is connected im kleinen at P .*

PROOF. Let R denote a region containing P , and let R_1 denote a region containing P which lies together with its boundary in R . There exists in R_1 a domain D containing P whose boundary is a subset of the sum of a finite number of continua T_1, T_2, \dots, T_n lying in

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† R. L. Moore, *Concerning simple continuous curves*, Transactions of this Society, vol. 21 (1920), pp. 313–320. Also R. L. Moore's *Foundations of Point Set Theory*, American Mathematical Society Colloquium Publications, vol. 13, New York, 1932; Theorem 20' of chapter 2 in particular. Hereinafter, this book will be referred to as *Foundations*.

‡ R. L. Wilder, *Concerning simple continuous curves and related point sets*, American Journal of Mathematics, vol. 53 (1931), pp. 39–55.

§ F. B. Jones, *Concerning the boundary of a complementary domain of a continuous curve*, this Bulletin, vol. 45 (1939), pp. 428–435.

|| See Axiom 5*₁ of my paper *Concerning certain topologically flat spaces*, Transactions of this Society, vol. 42 (1937), pp. 53–93.