ROOM ON DETERMINANTAL LOCI

The Geometry of Determinantal Loci. By T. G. Room. Cambridge, University Press; New York, Macmillan, 1938. 28+483 pp.

The original purpose of this volume, as stated in the preface, was to provide a systematic account, according to the method and scope of Reye's projective geometry, of the properties of the loci in higher space which are analogous to quadric and cubic surfaces of ordinary space. But these limitations were too narrow; in the final form, synthetic and algebraic methods are used freely, frequently combined in the same problem.

While part of the work is original with the author, and general account of recent contributions, especially of the English School, has been taken, the general scope and procedure of the book are those developed from three sources:

Baker, *Principles of Geometry*, 6 volumes;

Bertini, *Geometria Proiettiva degli Iperspazi*;

Segre, *Mehrdimensionale Räume* (Encyklopädie, III, C7).

Usually citations are not given to original sources, but to the treatment of the problem under consideration in one or another of the three treatises mentioned. Some important recent contributions are not cited at all.

The subject is introduced by concrete examples. Every algebraic plane curve can be defined by a determinant equated to zero. The eliminant of the parameters in two projective pencils of planes defines a quadric surface. The two systems of generators appear at once from its form. Similarly, the equation of a general cubic surface can be so written. Lines on the surface, properties of the double-six, and the plane representation are immediate consequences. Similarly, the space cubic curve is expressed by a two by three matrix of rank one; its systems of bisecants and of tangents are now apparent.

The notation and elaborate symbolism are explained at length. It is a curious fact that almost every locus discussed in the literature on algebraic geometry can be expressed by determinants. Many properties of one locus are shown to be projections or sections of another in a space of a different number of dimensions. Joins, meets, duals (in various spaces) have their usual meanings, and are employed freely, with an appropriate symbolism.

The three fundamental characteristics of any algebraic locus are its dimension, order, and its freedom. The symbol $(|p, q|_r, [n])$ is used to denote the locus of meets of the sets of corresponding linear spaces $[n-p+r]$ of q related stars $]p-1[$ in $[n]$. The symbol $|p, q|$ means a matrix of p rows, q columns, each element being a linear form in point coordinates in linear space of n dimensions. The number r is the rank of the matrix.

Each such locus can be generated in two ways by systems of projective primes (loci defined by one linear equation in point coordinates), and may have multiple loci of fewer dimensions. This establishes a (1, 1) correspondence among the points of one locus, or between the points of two different loci. Thus is opened a new vista of Cremona transformations. However, with few exceptions these are not further developed in the book. It does, on the other hand, use this scheme freely and with skill in developing the representation of a locus upon a linear space.

The plan of the book is to develop a few general theorems and then to elaborate the properties for particular values of p, q, r, n, and to show consequences of restricted