DECOMPOSITION OF ELEMENTS IN ABELIAN GROUPS*

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1. **Introduction.** Let G be an abelian group of elements g with operation $+$ (sum), and unit element 0 (zero). We shall be concerned with the following property:

PROPERTY P_n. *There exist n distinct elements in G such that their sum vanishes.*

In the present paper we determine necessary and sufficient conditions for an abelian group to have the property P_n. The author first proved the validity of these conditions for fields, but, as noted by T. Nakayama, the operation $\times$ does not occur, so that the theory may be stated for a system of elements with only one operation $+$ defined for these elements. Groups with the property P_n have useful algebraic applications.†

One is naturally led to consider the decomposability of any given element of a group G into a sum of distinct elements of G. This problem is treated in §3.

It is necessary in the treatment of decompositions of elements to distinguish only between nonzero elements of order 2, elements of order different from 2, and the zero element.

2. **Groups with property** P_n. A pair (h, k) of elements h, k in a group G satisfying

$$h + k = g$$

is called a *g-pair* in G. If one or both of the elements h, k is zero, the pair (h, k) is called a *null g-pair*.

An element q in G of order 2 $(q+q=0)$ will be said to be *singular*. The remaining elements of G are said to be *nonsingular*.

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† Let the minimal number $m(F)$ of a form F of degree r, with respect to a field K^*, designate the smallest value of σ for which F can be written as a sum $\lambda_1 L_1^r + \cdots + \lambda_\sigma L_\sigma^r$, where the λ's are in K^*, and the L's are linear forms with coefficients in K^*. Let K denote a field whose characteristic and order are such that the symmetric q-way and $(q+1)$-way matrices of the forms Q and QL of degree q and $(q+1)$, respectively, are unique, the coefficients being in K. We can prove that if K has the property P_{q+1}, the following inequality is true:

$$m(QL) \leqq (q+1)m(Q),$$

where the m's denote minimal numbers with respect to K.