of the form $4k+1$ *or both of the form* $4k+3$, *and is zero otherwise.*

THEOREM 5. *From* III$'$, $N_3(n\equiv 6)=24G'(n)$.

THEOREM 6. *From* VI$'$, $N_3(n\equiv 0)=8G'(n)+4H(n)-4e(n)$.*

THEOREM 7. *From* VI$'$, $N_3(n\equiv 4)=8G'(n)+4H(n)-8e(n)$.*

Since $n\equiv 7 \pmod 8$ cannot be represented as the sum of three squares, the set of formulas is complete.

It is clear that by selecting other functions $F(x, y, z)$ in a suitable manner other arithmetical results implicit in our general formulas may be obtained. As is usually the case in results of this type, strictly elementary proofs are no doubt possible, but are sometimes difficult to establish even after the theorems are known.

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INVOLUTORY SYSTEMS OF CURVES ON RULED SURFACES†

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In a paper presented to the International Mathematical Congress in Toronto in 1924, N. B. McLean discussed the properties of a certain one-parameter family of curves lying upon a ruled surface and characterized by the condition of forming a constant cross ratio with the complex curves of the surface. It is the purpose of the present paper to generalize McLean's system of curves and then to call attention to certain interesting special cases.

For the defining system of differential equations we make use of the form

$$\text{(R)} \qquad y'' + p_{12}z' + q_{11}y + q_{12}z = 0, \quad z'' + p_{21}y' + q_{21}y + q_{22}z = 0,$$

where $p_{12}'=2q_{12}$, $p_{21}'=2q_{21}$. For this form the two directrix curves C_y, C_z are the two branches of the flecnode curve of R.

The tetrahedron of reference is that determined by the four points P_y, P_z, P_ρ, P_σ, where

$$(1) \qquad \rho = 2y' + p_{12}z, \qquad \sigma = 2z' + p_{21}y,$$

the unit point being so chosen that the general point of space will be represented by the expression

$$x_1y + x_2z + x_3\rho + x_4\sigma.$$

† Presented to the Society, April 16, 1938.