

for constants. In *Principia* the propositional function vacillates vaguely between this status and the status of a property or relation "in intension," the latter status being the one relevant to the formal developments; but Tarski follows the current trend of dispensing with propositional functions in the latter sense in favor of ordinary classes and relations.

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Projektive Geometrie. By Karl Doehlemann. New one-volume edition by Heinrich Timerding. (Sammlung Götschen, no. 72.) Berlin and Leipzig, de Gruyter, 1937. 131 pp.

Earlier editions of this title in the Sammlung Götschen consisted of two parts, nos. 72 and 876, written by Karl Doehlemann. One familiar with them will find in the volume under review a new book, entirely rewritten and differing from its predecessors in size, content, and style. It goes by the same title, however, and bears the same number in the collection as Part I of former editions.

For example, the third edition of no. 72, published in 1912, contains 179 pages and an index, and is divided into seven sections of which the first six carry the reader through the pole-polar theory of conics, that is, through the elements of projective geometry, customarily so-called. The seventh section is devoted to cones and ruled surfaces of the second order.

The present volume, on the other hand, attempts a more ambitious program. This will be sufficiently indicated by a list of its chapters: 1. Projektive Grundgebilde in der Ebene; 2. Kurven zweiter Ordnung; 3. Projektive Geometrie des Raumes; 4. Flächen zweiter Ordnung; 5. Raumkurven dritter Ordnung; 6. Kollineationen und Korrelationen. There is no index, but the Inhaltsverzeichnis, in a volume of this size, seems to be sufficient as a guide to particular items.

The first two chapters (59 pages) are devoted to the projective geometry of the plane, swiftly developed in the synthetic manner with considerable appeal to intuition. The double ratio is introduced immediately after central projection and ideal elements (in the plane); and the projective relationship is defined in terms of equal double ratios. Then, after harmonic points have been defined as four points whose double ratio is -1 (when they are taken in a certain order), Desargues' Theorem for two coplanar triangles is given, and is followed by brief discussions of involutions, duality, and the projective ordering of points on a line. With this introduction, the theory of conics, Pascal's Theorem, and poles and polars are developed. Thus, after a short interlude on imaginary elements, the first and second chapters dispose of the material usually considered in an elementary course on projective geometry.

This more or less detailed examination of the two first chapters indicates the nature of the whole treatment. Everything is directed toward one end, to cover ground with the strictest economy of expression. Consequently, the arrangement of topics is found to be somewhat different from the usual arrangement; and the principle of duality is seldom invoked after its initial statement. We find, for example, Brianchon's Theorem, in the second chapter, discussed only as an illustration of the theory of polar reciprocals; and, in the fourth chapter, ruled surfaces are introduced as particular types of surfaces of the second order.

Whatever the disadvantages of such a rapid flight over the field (and I am inclined to feel that much of the essence of geometry is missed by such an enumeration of its facts only, without mentioning its logical implications), it does take one to unfrequented places. Thus I would note the fifth chapter, on space curves of third order,