ON THE REMAINDER IN THE APPROXIMATE EVALUATION OF THE PROBABILITY IN THE SYMMETRICAL CASE OF JAMES BERNOULLI'S THEOREM*

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1. *Introduction.* In this paper we consider the symmetrical case of James Bernoulli's theorem in the theory of probability. We let m represent the number of successes of an event in a series of n independent trials with constant probability $p = 1-q = 1/2$ for the success of each trial. Then we seek the probability P of the inequality

$$\left| m - \frac{n}{2} \right| \leqq \epsilon, \tag{1}$$

where ϵ is a given arbitrary positive number. The probability P is usually given by an approximate formula without mention of the error term or remainder involved.† In 1926, D. Mirimanoff‡ discussed this error term and gave results which are similar, but not as free from restrictions as those obtained here by entirely different methods.§

2. *The Exact Expression for P.* Let T_m represent the probability of m successes in the n trials and consider its generating function

$$\sum_{m=0}^{n} T_m t^m,$$

where t is an arbitrary variable. It has been shown‖ that

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† See, for example, I. Todhunter, *A History of the Mathematical Theory of Probability*, 1865, pp. 548–553.

‡ D. Mirimanoff, *Le jeu de pile ou face et les formules de Laplace et de J. Eggenberger*, Commentarii Mathematici Helvetici, vol. 2 (1926), pp. 133–168.

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‖ For this and similar results see A. A. Markoff, *Wahrscheinlichkeitsrechnung*, 1912, pp. 18–44.