ON THE GENERATION OF THE FUNCTIONS Cpq AND Np OF LUKASIEWICZ AND TARSKI BY MEANS OF A SINGLE BINARY OPERATION

BY J. C. C. McKINSEY*

Indicating the n "truth-values" of a Lukasiewicz-Tarski logic† by the n numbers $1, 2, \cdots, n$, we define the functions Cpq and Np as follows:

$$Cpq = 1, \text{ when } p \geqq q,$$
$$Cpq = q - p + 1, \text{ when } p < q,$$
$$Np = n - p + 1.$$

Thus, for example, for $n = 3$ we have

C	1	2	3
1	1	2	3
2	1	1	2
3	1	1	1

p	Np
1	3
2	2
3	1

I shall denote a Lukasiewicz-Tarski logic of n truth-values by L_n.

In this paper I define,‡ in terms of Cpq and Np, a function E_ipq such that, in each L_n, Cpq and Np are in turn definable in terms of $E_{n-2}pq$. The function E_ipq is defined by means of the following series of definitions.

DEFINITION 1. $A_0p = p, A_{i+1}p = CNpA_ip$.
DEFINITION 2. $B_0p = Np, B_{i+1}p = CpB_ip$.
DEFINITION 3. $D_ip = CA_ipNCpNB_ip$.
DEFINITION 4. $E_ipq = CpD_iq$.

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* Blumenthal Research Fellow.

† For a general discussion of this logic, see Lewis and Langford, *Symbolic Logic*, pp. 199–234.

‡ D. L. Webb has recently found (*The generation of any n-valued logic by one binary operation*, Proceedings of the National Academy of Sciences, vol. 21 (1935), pp. 252–254) a binary operation by means of which it is possible to generate any operation of any n-valued logic. His operation, however, cannot be defined in terms of Cpq and Np except when $n = 2$. This can be seen from the fact that the operations Cpq and Np are class-closing on the elements $1, n$; whereas the operation found by Webb has not this property.