SIMILARITY OF MATRICES IN WHICH THE ELEMENTS ARE REAL QUATERNIONS*

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1. *Introduction.* The purpose of this paper is to give a necessary and sufficient condition that for two matrices, A and B, of which the elements are real quaternions, there exist a non-singular matrix S whose elements are real quaternions such that $SAS^{-1}=B$. The matrices A and B are said to be similar if such a matrix S exists. This paper defines a set of invariant factors for any such matrix, A, in terms of the ranks of certain real polynomials in A.

2. *Definitions and Notations.* If A represents a matrix having m rows and n columns, then A' (read A transpose) is the matrix A with the rows and columns interchanged so that A' has n rows and m columns.

According to E. H. Moore a set of k vectors η_i, each being a matrix having n rows and one column, where $\eta_i' = (y_{i1}, y_{i2}, \cdots, y_{in})$, $(i=1, 2, \cdots, k)$, whose elements y_{ij} are real quaternions, is left linearly dependent with respect to real quaternions if there exists a set of constants q_i, which are real quaternions and not all zero such that $\sum_{i=1}^{i=k} q_i y_{ij}=0$, $(j=1, 2, \cdots, n)$. If no such set of real quaternions, q_i, exists except $q_i\equiv 0$, the vectors η_i are said to be left linearly independent. Similarly the k vectors η_i are right linearly dependent with respect to real quaternions if there exists a set of constants, q_i, which are real quaternions and not all zero, such that $\sum_{i=1}^{i=k} y_{ji} q_i=0$, $(j=1, 2, \cdots, n)$. If no such set of real quaternions, q_i, exists except $q_i\equiv 0$, the vectors η_i are said to be right linearly independent.

Moore considered the columns of a matrix, whose elements are real quaternions, as vectors and defined the rank, r, of such a matrix, S, as the maximum number of columns of S which are right linearly independent with respect to real quaternions. He proved that if a matrix S is of rank r, then r is also the maximum number of rows of S that are left linearly independent with

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