4. *Remark.* Let the sequence $E_1, E_2, \cdots$ be as in §2. Then we can even assert *that for every* $\lambda<1$ *there exists an infinite subsequence* $E_{i_1}, E_{i_2}, \cdots$ *such that for every* p *and* q

$$\mu(E_{i_p}E_{i_q}) \geqq \lambda m^2.$$

We show first that there exists an infinite subsequence $E_{k_1}, E_{k_2}, \cdots$ such that $\mu(E_{k_1}, E_{k_p}) \geqq \lambda m^2$ for every p. Suppose that no such subsequence exists; then to every $n=1, 2, \cdots$ belongs a p_n such that

$$\mu(E_nE_m) < \lambda m^2 \quad \text{for} \quad m \geqq n + p_n.$$

Writing $n_1=1$, $n_2=n_1+p_{n_1}$, $n_3=n_2+p_{n_2}$, $\cdots$, we have then for every i and k,

$$\mu(E_{n_i}E_{n_k}) < \lambda m^2,$$

which contradicts the theorem of §2. The proof is now easily completed by applying the diagonal principle.

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ON THE ZEROS OF THE DERIVATIVE OF A RATIONAL FUNCTION*

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1. *Introduction.* The primary object of this note is to give a simple solution of a problem already discussed by many authors including the present one.† It is the problem of determining the regions within which lie the zeros of the derivative of a rational function when the zeros and poles of the function lie in prescribed circular regions.

THEOREM 1.‡ *For* $j=0, 1, \cdots, p$ *let* r_j *and* σ_j *be real constants*

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* Presented to the Society, September 4, 1934.

† For an expository account and list of references see M. Marden, American Mathematical Monthly, vol. 42 (1935), pp. 277–286, hereafter referred to as Marden I.

‡ See M. Marden, Transactions of this Society, vol. 32 (1930), pp. 81–109, hereafter referred to as Marden II.