ASSOCIATED ALGEBRAIC AND PARTIAL DIFFERENTIAL EQUATIONS

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1. *Introduction*. Between an algebraic equation and an ordinary linear homogeneous differential equation with constant coefficients, in a single unknown, there is a familiar and useful association, illustrated by the equations

$$x^2 + 3x - 4 = 0, \qquad u'' + 3u' - 4u = 0.$$

Riquier* has shown that a similar association exists between finite systems consisting of m algebraic equations in n unknowns and m linear homogeneous partial differential equations in a single unknown and n independent variables as follows. Let

$$\sum_{0}^{p} \overset{\alpha}{a}_{i_1 \cdots i_n}(i_1 \cdots i_n) = 0, \qquad (\alpha = 1, \cdots, m), \tag{1}$$

be any system of algebraic equations, where the a's are constants and $(i_1 \cdots i_n)$ is to be interpreted as the monomial $x_1^{i_1} \cdots x_n^{i_n}$. Let the associated system of partial differential equations be

$$\sum_{0}^{p} \overset{\alpha}{a}_{i_1 \cdots i_n}(i_1 \cdots i_n)u = 0, \qquad (\alpha = 1, \cdots, m), \tag{2}$$

where $(i_1 \cdots i_n)$ is to be interpreted as the differential operator $\partial^{i_1 + \cdots + i_n}/\partial x_1^{i_1} \cdots \partial x_n^{i_n}$.

We shall prove in this paper the two following theorems.

THEOREM 1. *System* (1) *is inconsistent if and only if the general solution of* (2) *is* $u=0$.

THEOREM 2. *The general solution of* (2) *is a non-zero polynomial if and only if* $x_1 = \cdots = x_n = 0$ *is the solution of* (1).

2. *Corresponding Operations on the Two Systems*. We shall mul-

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* *Sur la résolution numérique du système d'équations algébriques entières à un nombre quelconque d'inconnues*, Annales Scientifiques de l'École Normale Supérieure, vol. 63 (1928), pp. 145–188.