SOME MULTIPLICATION THEOREMS FOR THE NÖRLUND MEAN

BY FLORENCE M. MEARS

Absolute summability for the series $\sum_{n=1}^{\infty} u_n$ by the Cesàro mean and by the Riesz mean have been defined by Fekete* and by Obrechkoff,† respectively. In each case, theorems for the multiplication of series summed by these means have been proved.‡ The purpose of this paper is to establish a definition for absolute summability by the Nörlund mean, and to prove three multiplication theorems for this mean. Theorem 1 includes Mertens' theorem for convergent series and its extension for the Cesàro mean. Theorem 2 includes Cesàro's multiplication theorem. Theorem 3 includes the following theorem by M. J. Belinfante for the Cesàro mean.

If $\sum_{n=1}^{\infty} u_n$ is summable C_s to U, and if $\sum_{n=1}^{\infty} v_n$ is summable C_r to V, and bounded C_{r-1}, $(s \geqq 0, r \geqq 1)$, the product series $\sum_{n=1}^{\infty} w_n$ is summable C_{r+s} to UV.§

For any given series $\sum_{k=1}^{\infty} u_k$, with terms real or complex, form the sequence $\{U_k\}$, where $U_k = \sum_{n=1}^{k} u_n$. Let $\{a_n\}$ be a sequence of positive numbers, and let $A_k = \sum_{n=1}^{k} a_n$. The series $\sum_{k=1}^{\infty} u_k$ is said to be summable to U' by the Nörlund mean A if

$$\lim_{n \to \infty} U_n' = \lim_{n \to \infty} \frac{\sum_{k=1}^{n} a_{n-k+1} U_k}{A_n}$$

exists and is equal to U'.¶ If $\sum_{k=1}^{\infty} u_k'$, where $u_n' = U_n' - U_{n-1}'$, is absolutely convergent, we shall say that $\sum_{k=1}^{\infty} u_k$ is absolutely summable A. We shall assume that $\lim_{n \to \infty}(a_n/A_n) = 0$; then A is a regular method of summation.‖

* Matematikai és Természettudományi Értesitö, vol. 32 (1914), pp. 389–425.

† Comptes Rendus, vol. 185 (1928), pp. 215–217.

‡ For discussion and references, see Kogbetliantz, Mémorial des Sciences Mathématiques, No. 51.

§ Koninklijke Akademie te Amsterdam, Verslag, vol. 32 (1923), pp. 177–189.

¶ Riesz, Proceedings of the London Mathematical Society, (2), vol. 22 (1923), p. 412.

‖ Riesz, loc. cit.