ON THE COMPOSITION OF QUADRATIC FORMS*

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1. *Introduction.* A compound of two binary quadratic forms $f_1(x_1, y_1)$ and $f_2(x_2, y_2)$ is a third form $f(x, y)$ such that $f(x, y) = f_1(x_1, y_1)f_2(x_2, y_2)$ under a primitive bilinear transformation

$$x = p_1x_1x_2 + p_2x_1y_2 + p_3y_1x_2 + p_4y_1y_2,$$
$$y = q_1x_1x_2 + q_2x_1y_2 + q_3y_1x_2 + q_4y_1y_2.$$

The fundamental problem of this theory is to determine a compound of two given forms and the transformation under which the relationship exists.

This problem has been considered† by Gauss, Arndt, Dedekind and others. The method of Dedekind‡ was based upon a correspondence between forms and moduls in an algebraic field, composition of forms corresponding to multiplication of moduls.

The method of the present paper is also based upon a correspondence between forms and moduls. These moduls form a subclass of those considered by Dedekind in that only integral moduls are employed. These moduls in turn are in correspondence with matrices with rational integral elements, the greatest common divisor process corresponding to multiplication of moduls and hence to composition of forms. By this correspondence and the matric g. c. d. process of Châtelet,§ the composition of quadratic forms requires but a small fraction of the time and labor required by previously known methods.

2. *The Correspondence.* Let $1, \theta$ be an integral basis for the quadratic field $\mathfrak{F}(\sqrt{m})$. Then $\theta^2 = m$ if $m \equiv 2, 3 \pmod 4$, (Case 1); and $\theta^2 = \theta + m', 4m' = m - 1$ if $m \equiv 1 \pmod 4$, (Case 2). Let $a, b,$ and k be rational integers, of which a and k are positive, such that $b^2 - k^2 \equiv 0 \pmod a$, (Case 1); $b^2 + bk - k^2m' \equiv 0 \pmod a$, (Case 2).

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† Gauss, *Disquisitiones Arithmeticae*, 1801, §234 ff.; Arndt, *Auflösung einer Aufgabe in der Composition der quadratischen Formen*, Journal für Mathematik, vol. 56 (1859), pp. 64–71.

‡ Dirichlet, *Vorlesungen über Zahlentheorie*, 4th ed., 1894, pp. 640–657.

§ A. Châtelet, *Groupes Abéliens Finis*, 1924, p. 26.