

THE RELATIVE CONNECTIVITIES OF SYMMETRIC PRODUCTS*

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1. *Introduction.* The topology of the domain of discontinuity of a finite group of transformations operating on a complex, and, in particular, the topology of symmetric product complexes, has been studied by P. A. Smith[†] and the author.[‡] Following a suggestion made by Morse,[§] we obtain in this note explicit formulas for the so-called relative connectivities of the symmetric product of a complex in terms of its mod 2 Betti numbers, and we discuss an application of this result to the theory of critical chords. First, however, we derive a more general result of which the formulas for the relative connectivities of symmetric products is a special case. The methods used here follow closely those of S.

2. *Definitions and Preliminary Theorems.* For proofs or fuller discussion of statements made in this section, the reader is referred to S or R.

Let K be a simplicial n -complex.|| Let T be a topological involution such that (a) T carries m -simplexes of K into m -simplexes of K ; (b) if a simplex of K is invariant, it is pointwise invariant.

The invariant simplexes of K form a subcomplex K^0 , and the non-invariant simplexes can be grouped in pairs so that each member of a pair is transformed into the other member by T . Thus the m -simplexes of K can be renamed $E_m^i, \bar{E}_m^i, E_m^{0j}$, where $\bar{E}_m^i = TE_m^i$, and E_m^{0j} is a simplex of K^0 . If ¶ $C = t_i E_m^i$ is a chain of

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† P. A. Smith, *The topology of involutions*, Proceedings of the National Academy of Sciences, (1933), pp. 612–618. (Denoted hereafter by S.)

‡ M. Richardson, *On the homology characters of symmetric products*, Duke Mathematical Journal, vol. 1 (1935), pp. 50–69. (Denoted hereafter by R.)

§ M. Morse, *The Calculus of Variations in the Large*, Colloquium Publications of this Society, vol. 18, 1934, p. 191. (Denoted hereafter by M.)

|| Our general topological terminology and notation is that of S. Lefschetz, *Topology*, Colloquium Publications of this Society, vol. 12, 1930.

¶ A repeated index indicates summation.