ON THE THEORY OF RESIDUES OF POLYGENIC FUNCTIONS*

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1. *Introduction.* In polygenic function theory we are interested in a sub-class of the class of all functions, such that

$$f(z) = f(\xi, \zeta),$$

where ξ and ζ are complex variables; that is, such that when ξ and ζ are assigned $f(z)$ is known. The particular sub-class to which we restrict ourselves is the class such that ζ is always the conjugate of ξ, or

$$f(z) = f(z, \bar{z}).$$

For a brief outline of this subject and a quite complete bibliography one should consult the paper by Hedrick† in this Bulletin.

It is the purpose of this paper to generalize the definitions for residues of polygenic functions previously given‡ and to extend the theory. Incidentally in the process, the circulation theorems§ are generalized; a theorem on residues of regular functions is obtained, while the theory is applied to the large class of functions defined by a Laurent series.

2. *The Definitions for Residues.* If we surround the point $z=a$ by a circle O, center at a and radius r, then the residue R_z of $f(z)$ is defined by the equation,‖

$$(1) \qquad R_z = \operatorname*{limit}_{r\to 0} \frac{1}{2\pi i}\int_O f(z)dz;$$

while the residue $R_{\bar{z}}$, which is of equal importance, is

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† E. R. Hedrick, *Non-analytic functions of a complex variable*, this Bulletin, vol. 39 (1933), pp. 75–96.

‡ V. C. Poor, *Residues of polygenic functions*, Transactions of this Society, vol. 32 (1930), pp. 216–222.

§ Poor, loc. cit. Calugaréano, (Thesis), *Sur les fonctions polygènes d'une variable complexe*, 1928, p. 11.

‖ Poor, loc. cit., §1.