STONE ON HILBERT SPACE

Linear Transformations in Hilbert Space and their Applications to Analysis. By M. H. Stone. New York, American Mathematical Society, 1932. viii +622 pp.

The attitude of mathematicians towards the applications varies. Some feel a need of rationalizing their personal interests by pointing out their importance for the applications, and the applications know them not. Others assert with pride that their results cannot possibly be applied, and fate whimsically harnesses their purest dreams to the chariot of industry. Whether we want our results to be applied or not, we probably all agree that the applications have a way of posing stimulating questions which spur the progress of our science. Thus we probably owe more of our advance in analysis to the prying curiosity of the physicist than to any other agent.

We recall how the problem of wave motion and heat conduction led to the development of the function concept and the introduction of orthogonal series, basic elements of present day analysis. Dirichlet's problem in potential theory had a profound influence on the calculus of variations and led to Fredholm's theory of integral equations. We know what happened to this theory in the masterly hands of Hilbert, how it became a theory of orthogonal transformations and reduction of quadratic forms, how Hilbert revived the interest in orthogonal functions, and created the atmosphere which stimulated, for example, F. Riesz in his basic discoveries of different types of convergence in function spaces. One is perhaps justified in regarding F. Riesz's *Systèmes Linéaires d'une Infinité d'Inconnues* as the climax of the development of this period, though its novel points of view really mark it off as the forerunner of much of the modern theory.

It is only fair to admit that much of this development seemed of little interest to the average physicist who, of course, knew that Dirichlet's problem could be solved long before we actually solved it. But there was heavy weather ahead for the physicists; the foundations of physics were taken for a ride, and a bewildered physicist had to reorient himself in a strange world, perhaps not created by a mathematician, but at least one where the mathematician played the role of the little tin god on wheels. Space and time became differential geometry and tensor analysis. Energy and matter disappeared in a cloud of operators, abstract algebra, probabilities, and boundary value problems.

The analytical problems of quantum and wave mechanics can be thought of in terms of linear transformations in a space of infinitely many dimensions with certain specified properties, the abstract Hilbert space. To Hilbert and his pupils we owe a theory of completely continuous and of bounded transformations of this space. An excellent account of this theory is to be found in the Encyclopädie article of Hellinger and Toeplitz. The modern theory of unbounded transformations is connected chiefly with the names of T. Carleman, J. von Neumann, F. Riesz, M. H. Stone, and A. Wintner. Carleman's work on integral equations started in 1916; a powerful analytical technique enabled him to push this theory far beyond the bounds reached by the Hilbert