THE SUMMATION OF SERIES OF ORTHOGONAL POLYNOMIALS*

BY DUNHAM JACKSON

1. *Introduction.* The convergence of series of orthogonal polynomials can be treated under very broad hypotheses as to the character of the weight function with respect to which the property of orthogonality holds, if compensating restrictions are placed on the function whose series development is under consideration.† It is to be shown here that certain questions of summability lend themselves to a comparable treatment, simple as to demonstration and in some ways perhaps crude as to results, but characterized by broad generality in the hypotheses on the weight function. The discussion of summability, unlike the particular theory of convergence to which reference has been made, depends on the representation of the partial sum of the series by means of the Christoffel-Darboux identity. The method will be illustrated first under simple conditions, which will then be seen to admit generalization in various directions.

The problem of trigonometric approximation, as usual, is similar in principle but somewhat different in detail.

2. *Application of Schwarz's Inequality to the First Arithmetic Mean.* Let $p_0(x)$, $p_1(x)$, $\cdots$ be the system of normalized orthogonal polynomials for the interval $(-1, 1)$ corresponding to the summable non-negative weight function $\rho(x)$, let a function $f(x)$ (always supposed to be such that ρf is summable, and to be subjected to further restrictions later) be expanded in series of these polynomials, and let $s_n(x)$ be the partial sum of the series through terms of the nth degree. Let

$$\sigma_n(x) = \frac{1}{n}\sum_{k=0}^{n-1} s_k(x).$$

The sums $s_k(x)$ and the corresponding errors of approximation $s_k(x)-f(x)$ can be represented in compact form by use of the

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† See, for example, the writer's paper entitled *Certain problems of closest approximation*, this Bulletin, vol. 39 (1933), pp. 889–906.