ON A CLASS OF EXISTENCE THEOREMS IN DIFFERENTIAL GEOMETRY

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1. *Introduction.* Many problems in differential geometry require the integration of a mixed system of differential equations

$$(\mathrm{E}_1) \qquad \frac{\partial u^\alpha}{\partial x^i} = \overset{\alpha}{\psi}_i(u, x),$$

$$(\mathrm{E}_2) \qquad F_0(u, x) = 0,$$

which gives rise to an infinite sequence of sets of algebraic equations

$$(1) \qquad F_1 = 0, F_2 = 0, \cdots, F_N = 0, \cdots,$$

related to the system E by the following property.

A necessary and sufficient condition for the existence of a solution of the system E *is that there exist an integer N such that the first N sets of equations of the sequence* (1) *be algebraically consistent and that all their solutions satisfy the* $(N+1)$*st set of equations of the sequence.*

In many cases the functions ψ and F_0 are linear and homogeneous in the unknowns u with the result that the equations (1) are of a similar character; also the coefficients A of the unknowns in (1) are the components of tensor differential invariants. In fact, the left members of each set of the equations (1) break up into the components of scalars and tensors. We shall confine our attention to systems E of this type. Since such systems E always possess a trivial solution $u=0$, we shall mean by a solution $u(x)$ of E a non-trivial solution of this system.

A necessary and sufficient condition for the existence of a solution of the first N sets of equations of the sequence (1) is the vanishing of their resultant system $R_N(A)$. For equations of the type under consideration in which the coefficients are real quantities, the solution whose existence is implied by the vanishing of the resultant system will be real.*

* When the equations (1) are linear and homogeneous in the unknowns u^α, the resultant system R_N can be taken to consist of the totality of all determinants of order L, equal to the number of unknowns u^α, which can be formed from the matrix of the coefficients of the first N sets of equations (1).