

SIMPLIFICATION OF THE WHITEHEAD-HUNTINGTON SET OF POSTULATES FOR THE ALGEBRA OF LOGIC

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1. *Introduction.* In 1916, B. A. Bernstein† obtained a reduction in the number of postulates of the Whitehead-Huntington set‡ for the algebra of logic. In this paper I secure a further reduction in the number of the postulates by combining the commutative and distributive laws and then proceed to investigate the postulates which result when the two reductions are effected at once.

I prove the sufficiency of the new postulates for the algebra of logic by showing that the original set can be derived from them. Finally I establish the independence of the new postulates by furnishing proof-systems of a simple arithmetic type.

2. *The New Postulates.* The new postulates have as undefined ideas the undefined ideas of the original set, namely, a *class*  $K$  and two *binary operations*  $+$  and  $\times$ . The postulates are the Postulates 1–6 listed below. In Postulates 3 and 4 is implied the condition *if the elements involved and their indicated combinations belong to  $K$* .

POSTULATE 1.  $a+b$  is in  $K$  whenever  $a$  and  $b$  are in  $K$ .

POSTULATE 2.  $ab$  is in  $K$  whenever  $a$  and  $b$  are in  $K$ .

POSTULATE 3.  $bc+a=(c+a)(b+a)$ .

POSTULATE 4.  $(b+c)a=ca+ba$ .

POSTULATE 5. For any element  $b$  in  $K$  there exists an element  $b'$  such that, whatever  $a$  is,  $a+bb'=a$  and  $a(b+b')=a$ .

POSTULATE 6. There are at least two elements,  $a$  and  $b$ , in  $K$  such that  $a \neq b$ .

3. *Sufficiency of the Postulates. Derivation of the Whitehead-Huntington Postulates.* The Whitehead-Huntington postulates leave undefined a *class*  $K$  and two *binary operations*  $+$  and  $\times$ ,

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† See this Bulletin, vol. 22 (1916), pp. 458–459.

‡ See the Transactions of this Society, vol. 5 (1904), pp. 288–309. The set referred to is the first of the three sets which occur in Huntington's paper.