NOTE ON CARSON'S INTEGRAL EQUATION

BY J. J. SLADE, JR.

The theory of Heaviside's operational methods has been developed in various papers. Wiener* has perhaps given the most rigorous theory of these operators. Carson† has certainly given the most elaborate.

For the manipulation of these operators Carson's methods appear to be the most direct. The solution of the operational equation

$$f(p) = \phi(t)$$

is identified with the solution of the integral equation‡

$$f(p) = p\int_0^\infty \phi(t)e^{-pt}dt, \tag{1}$$

in which the real part of p is positive, the path of integration is along the real axis, and $\phi(t)$ is to be defined for positive real values of t.

Dalzell§ has pointed out that the general solution of the equation is

$$\phi(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{f(p)}{p}e^{pt}dp,$$

and March|| has shown that Bromwich's contour integrals satisfy it.

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* *The operational calculus*, Mathematische Annalen, vol. 95 (1925), pp. 557–584.

† *The Heaviside operational calculus*, this Bulletin, vol. 32 (1926), pp. 43–68; *Electric Circuit Theory and the Operational Calculus*, McGraw-Hill, 1926; *Asymptotic solution of an operational equation*, Transactions of this Society, vol. 31 (1929), pp. 782–792.

‡ Carson, this Bulletin, loc. cit.

§ *Heaviside's operational method*, Proceedings Physical Society, vol. 42 (1930), pp. 75–81.

|| *The Heaviside operational calculus*, this Bulletin, vol. 33 (1927), pp. 311–318. For Bromwich's theory, see Jeffreys, *Operational Methods in Mathematical Physics*, Cambridge Tract No. 23, 1931, Chapter 2.