ON CUBIC CONGRUENCES*

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1. *Introduction.* In the consideration of metabelian groups G of order p^{n+3} which contain a given abelian group H of order p^n and type $1, 1, \cdots$, there enters an irreducible cubic congruence

$$(1) \qquad x^3 + \gamma x^2 - \alpha x + \beta \equiv 0, \pmod{p}.$$

It is necessary to determine how many congruences (1) are distinct under certain transformations on the generators of G.†

Let the generators of H be $s_1, s_2, \cdots, s_n$, and let U_1, U_2, U_3 be three operators of order p from the group of isomorphisms of H. Let the operators $s_1, \cdots, U_3$ be permutable except for the relations

$$(2) \qquad \begin{aligned} U_1^{-1}s_1U_1 &= s_1s_3, & U_2^{-1}s_1U_2 &= s_1s_5, & U_3^{-1}s_1U_3 &= s_1s_3^{\alpha}s_4^{\beta}s_5^{\gamma}, \\ U_1^{-1}s_2U_1 &= s_2s_4, & U_2^{-1}s_2U_2 &= s_2s_3, & U_3^{-1}s_2U_3 &= s_2s_5. \end{aligned}$$

Such operators U_1, U_2, U_3 obviously exist. The condition that $\{U_1, U_2, U_3\}$ contain no operator permutable with any operator, except identity, of $\{s_1, s_2\}$ is readily seen to be that (1) be irreducible, (mod p).

The group $G = \{H, U_1, U_2, U_3\}$ is a subgroup of the holomorph of H. For the sake of simplicity in the subsequent computations we shall show that generators of G may be chosen so that $\gamma = 0$, provided $p > 3$. Let $s_1' = s_1s_2^{-1}$, $U_2' = U_1^{-1}U_2$, and $U_3' = U_1U_2^{-2}U_3$. The operators $s_1', s_2, \cdots, s_n, U_1, U_2', U_3'$ generate G, and satisfy (2) with new numbers α', β', γ', where

$$\begin{aligned} \alpha' &= 2\gamma + \alpha - 3, \\ \beta' &= \alpha + \beta + \gamma - 1, \\ \gamma' &= \gamma - 3. \end{aligned}$$

Hence by repeating this transformation we may reduce γ to

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† See my paper, *On the metabelian groups which contain a given group H as a maximal invariant abelian subgroup*. This paper has been offered to the American Journal of Mathematics.