NORMAL DIVISION ALGEBRAS OVER ALGEBRAIC NUMBER FIELDS NOT OF FINITE DEGREE*

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1. *Introduction.* If R is the field of all rational numbers and if $\xi_1, \cdots, \xi_n$ are ordinary algebraic numbers, then the field $\Omega = R(\xi_1, \cdots, \xi_n)$ of all rational functions with rational coefficients of $\xi_1, \cdots, \xi_n$ is an *algebraic number field of finite degree* (the maximum number of linearly independent quantities of Ω) *over* R. It has recently been proved† that every normal *simple* algebra over such a field Ω is cyclic. In particular it has been shown that every normal *division* algebra of order n^2 (degree n) over Ω is cyclic and has exponent n.

In the present note I shall give an extension of the above results to *normal division algebras over any algebraic number field* Λ. I shall prove that all normal division algebras over Λ are cyclic and with degree equal to exponent but shall give a trivial example showing that the theorem corresponding to the above on normal *simple* algebras is false. The problem of the equivalence of normal division algebras over Λ will also be discussed.

2. *Cyclic Algebras.* Let F be any non-modular field and let Z by cyclic of degree n over F. Then Z possesses a generating automorphism

$$S: \qquad z \longleftrightarrow z^S, \qquad (z \text{ in } Z,\ z^S \text{ in } Z),$$

such that every automorphism of Z is one of $S^0 = S^n = I$, S, $S^2, \cdots, S^{n-1}$. The algebra A of all quantities

$$\sum_{i=0}^{n-1} z_i y^i, \qquad (z \text{ in } Z),$$

is a cyclic algebra with multiplication table

$$y^n = \gamma \text{ in } F,\ y^e z = z^{S^e} y^e, \qquad (e = 0, 1, \cdots),$$

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† See the paper by H. Hasse and myself in the Transactions of this Society, vol. 34 (1932), pp. 722–726, for the normal division algebra theorem. The theorem for normal simple algebras follows from Hasse's Theorem 6 of his Transactions paper, vol. 34 (1932), pp. 171–214.