ON UNIT-ZERO BOOLEAN REPRESENTATIONS OF OPERATIONS AND RELATIONS*

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1. *Introduction.* Consider an algebra $(K, +, \times)$, such as ordinary real algebra, in which there are two elements "0" and "1" having the properties that, for any element a,

$$(1) \qquad a + 0 = 0 + a = a,\ a1 = 1a = a.$$

Let

$$(x_1, x_2, \cdots, x_m; a_1, a_2, \cdots, a_m)$$

denote a *unit-zero function with respect to the sequence of m elements*, $a_1, \cdots, a_m$ of K, that is, a function $f(x_1, x_2, \cdots, x_m)$ of m elements $x_1, x_2, \cdots, x_m$ such that $f=1$ or 0, according as the equalities, $x_i = a_i$, $(i=1, 2, \cdots, m)$, all hold or do not all hold. Accordingly, $(x; a)$ will denote a *unit-zero function with respect to* a, that is, a function $f(x)$ such that $f(x)=1$ or 0, according as $x=a$ or $x \neq a$. Then the following propositions (2)–(4) evidently hold:

$$(2) \ (x_1, x_2, \cdots, x_m; a_1, a_2, \cdots, a_m) = (x_1; a_1)(x_2; a_2) \cdots (x_m; a_m);$$

$$(3) \qquad a(x_1, x_2, \cdots, x_m; a_1, a_2, \cdots, a_m) = a \text{ or } 0,$$

according as $x_i = a_i$, $(i=1, 2, \cdots, m)$, all hold or do not all hold;

$$(4) \ a(x_1, x_2, \cdots, x_m; a_1, a_2, \cdots, a_m) + b(x_1, x_2, \cdots, x_m; b_1, b_2, \cdots, b_m) = a, \text{ or } b, \text{ or } 0,$$

according as $x_i = a_i$ all hold, or $x_i = b_i$ all hold, or neither $x_i = a_i$ all hold nor $x_i = b_i$ all hold, $(i=1, 2, \cdots, m; a_i \neq b_i$ for some $i)$. In a previous paper† propositions (1)–(4) were made the basis of a method of obtaining arithmetic representations of arbitrary operations and relations in a finite class of elements. Since

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† B. A. Bernstein and N. Debely, *A practical method for the modular representation of finite operations and relations*, this Bulletin, vol. 38 (1932), pp. 110–114.