ON THE COMPACTNESS OF THE SPACE L_p

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1. *Introduction.* Let R_n be the n-dimensional euclidean space and $L_p (p>1)$ the function-space consisting of all the functions $f(x)$ defined and measurable over R_n, and such that the integrals

$$\int_{R_n} |f(x)|^p dx < \infty,$$

while the metric of the space L_p is defined, as usual, by the "distance"

$$\|f\| \equiv \left[\int_{R_n} |f(x)|^p dx \right]^{1/p}.$$

All the notions of *boundedness*, *convergence*, *limits*, *approximations*, etc., used in this note will be relative to this metric, unless explicitly specified to the contrary.

Let $S(x, \epsilon)$ be the n-dimensional sphere with center at x and radius ϵ. We designate by $V(\epsilon)$ the volume of $S(x, \epsilon)$, and by

$$f_\epsilon(x) \equiv \frac{1}{V(\epsilon)} \int_{S(x,\epsilon)} f(y) dy \tag{1}$$

the *moving average* of $f(x)$. Finally, let C_N be the n-dimensional cube, with center at the origin and edges of length $2N$ parallel to the coordinate axes. We set

$$f^N(x) = \begin{cases} f(x) \text{ when } x \text{ is in } C_N, \\ 0 \text{ elsewhere.} \end{cases} \tag{2}$$

A set $\mathfrak{F}$ of elements of L_p is called *compact* if every subset of $\mathfrak{F}$ contains at least one convergent sequence. In a recent paper Kolmogoroff† has derived necessary and sufficient conditions in order that $\mathfrak{F}$ be compact, under the restriction that all the elements of $\mathfrak{F}$ vanish outside of a fixed bounded measurable point

* Presented to the Society, October 31, 1931.

† Göttinger Nachrichten, 1931, pp. 60–63.