

For, the given identity implies

$$a_{2r} + b_{2r} + \cdots + c_{2r} = 0, \quad (r = 0, 1, \cdots);$$

and therefore

$$\begin{aligned} p_0(a_0 + b_0 + \cdots + c_0) + p_2(a_2 + b_2 + \cdots + c_2) + \cdots \\ + p_{2s}(a_{2s} + b_{2s} + \cdots + c_{2s}) = 0; \end{aligned}$$

which is the stated conclusion.

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ON SYMMETRIC PRODUCTS OF TOPOLOGICAL SPACES*

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1. *Introduction.* This paper is devoted to an operation that is defined for an arbitrary topological† space E and is analogous to the operation of constructing the combinatorial product spaces.‡ We shall be concerned with the topological properties of point sets defined by means of the above operation when executed on the segment $0 \leq x \leq 1$.

Let E be an arbitrary topological space. Let E^n denote the n th topological product of the space E , that is, the space whose elements are ordered systems (x_1, x_2, \cdots, x_n) of points $x_i \in E$. By a *neighborhood* of a point (x_1, x_2, \cdots, x_n) , we understand the set of all systems $(x'_1, x'_2, \cdots, x'_n)$, where x'_i belongs to a neighborhood u_i of the point x_i in the space E .‡

The operation with which we are concerned in this paper consists in constructing a space which we shall call the *n th symmetric product* of the space E and denote by $E(n)$. Its elements are *non-ordered* systems of n points (which may be different or not) belonging to E . Two systems differing only by the order or multiplicity of elements are considered identical. A non-ordered system or simply a *set* consisting of n points x_1, \cdots, x_n from the space E will be denoted by $\{x_1, x_2, \cdots, x_n\}$. If u_i is a neighborhood of the point x_i in the space E , then the set of all systems

* The definition of *symmetric products* is given below.

† In the sense of Hausdorff, *Grundzüge der Mengenlehre*, p. 228.

‡ See, for example, F. Hausdorff, *Grundzüge der Mengenlehre*, p. 102.