QUADRATIC PARTITIONS—I

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1. *Introduction.* This is the preliminary and longest note of a series which, by the kindness of the editors of this Bulletin, I hope to publish from time to time, giving some of the numerous general arithmetical theorems of a particular type which I have been accumulating for several years. To make this series self contained, I first recall the necessary definitions, and give once for all a few formulas that will be used repeatedly. Results and methods of two previous papers are indicated by numbered references.*

Subsequent notes will contain only theorems, with statements of the elementary identities from which they follow. This will be sufficient to enable anyone who wishes to retrace the details of the proofs and verify the conclusions. I believe that present conditions of mathematical publication in this country demand the utmost brevity consistent with reasonable clarity.

2. *Parity.* Let $\xi=(x_1, \cdots, x_n)$ be a one-row matrix or vector in which the *elements* $x_1, \cdots, x_n$ are in a given field K. Write $-\xi\equiv(-x_1, \cdots, -x_n)$. If $f(\xi)$ is a single finite real or complex number whenever $x_1, \cdots, x_n$ are in K, we say that $f(\xi)$ is *uniform over* K. Let $f(\xi)$ be uniform over K. Then, if $f(-\xi)=f(\xi)$, we say that $f(\xi)$ has *parity* $p(n\,|\,)$ in ξ; if $f(-\xi)=-f(\xi)$, and if further $f(0, \cdots, 0)=0$, the parity is $p(\,|\,n)$. Let us denote by $\xi_i, \eta_j, (i=1, \cdots, r; j=1, \cdots, s)$, vectors in K, having no element in common. Then, if $f(\xi_1, \cdots, \xi_r, \eta_1, \cdots, \eta_s)$ has parity $p(n_i'\,|\,)$ in ξ_i, and parity $p(\,|\,n_j'')$ in η_j $(i=1, \cdots, r; j=1, \cdots, s)$, we shall agree to say that $f(\xi_1, \cdots, \xi_r, \eta_1, \cdots, \eta_s)$ has parity $p(n_1', \cdots, n_r'\,|\,n_1'', \cdots, n_s'')$ in $(\xi_1, \cdots, \xi_r\,|\,\eta_1, \cdots, \eta_s)$, and we write

* (1) *Arithmetical paraphrases*, Transactions of this Society, vol. 22 (1921), pp. 1–30; 198–219; (2) *A revision of the Bernoullian and Eulerian functions*, this Bulletin, vol. 28 (1922), pp. 443–450. The material in (1) is included and generalized in (3) *Algebraic Arithmetic*, American Mathematical Society Colloquium Publications, vol. 7, 1927, Chapters 2, 3; ibid., pp. 146–159, contain a complete account of the umbral calculus used in (2) and in some of the present notes.