THE DISTRIBUTIVE LAWS FOR HOMOGENEOUS LINEAR SYSTEMS

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The terminology of addition and of multiplication may be readily extended to apply to homogeneous linear systems or to their geometrical interpretations, linear projective spaces. The product ab of two linear spaces a and b, denotes the intersection or greatest linear space common to the two given spaces. It is thus the logical product of the spaces. The sum $a+b$ denotes the join or least linear space containing the given spaces as subspaces. It is analogous to the logical sum but is distinguished from it by the fact that $a+b$ will in general contain also points which are in neither a nor b. Addition is commutative as is also multiplication. One has $a+a=a$, and $aa=a$, etc. Many of the formulas established for the algebra of logic hold here also.*

An essential feature of the algebra of logic is the following pair of relations, the first of which is applicable to ordinary algebra:

$$\text{I} \qquad a(b + c) = ab + ac,$$

$$\text{II} \qquad a + bc = (a + b)(a + c).$$

If I holds a is said to be distributive with respect to b and c in the first sense, if II holds, in the second sense. The proofs of these when based upon other postulates are not trivial.† For the case of even a one-dimensional projective space, these distributive relations fail to hold. Indeed if a, b, and c denote three distinct points of a line, and 0 denotes the null space, we have $b+c=$ entire line, $a(b+c)=a$, but $ab=0$, $ac=0$, $ab+ac=0$. Hence $a(b+c)\neq ab+ac$. Similarly $a+bc=a$, but $(a+b)$ $(a+c)$ $=$ entire line.

While from two elements a and b, one can generate by addition and multiplication only the closed set ab, a, b, $a+b$, yet

* See A. A. Bennett, *Semi-serial order*, American Mathematical Monthly, vol. 37 (1930), pp. 418–423.

† See E. V. Huntington, *Postulates for the algebra of logic*, Transactions of this Society, vol. 5 (1904), pp. 288–309, particularly Peirce's proof, pp. 300–302.