ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

172. Professor R. L. Jeffery: *Volterra integral equations of the first kind.*

This paper deals with a study of the equation (1) $f(x)=\int_0^x K(x,\xi)u(\xi)d\xi$ under the fgllowing conditions: (a) $f(0)=0$, $K(x,\xi)$ integrable in ξ for each x, $\{f(x)-f(x')\}/(x-x')$ and $\{K(x,\xi)-K(x',\xi)\}/(x-x')$ bounded; (b) the left hand derivatives $f'^{-}(x)$ and $K_x^{-}(x,\xi)$ respectively of $f(x)$, and of $K(x,\xi)$ with respect to x, existing; (c) conditions on $\phi(x, x')=\{\int_{x'}^{x} K(x,\xi)d\xi\}/(x-x')$. If (a) holds and $\phi(x,x')$ is bounded uniformly from zero for x' sufficiently close to x, then functions $u_\epsilon(x)$ such that $|f(x)-\int_0^x K(x,\xi)u_\epsilon(\xi)d\xi|<\epsilon$ exist for any given ϵ. If (a) and (b) hold, and if $\lim_{x'\to x}\phi(x,x')=\rho(x)\times 0$, then (1) has a unique bounded solution given by a series involving $f'^{-}(x)$, $K_x^{-}(x,\xi)$, and $\rho(x)$. If $\rho(x)\equiv 0$, let $\rho_1(x)$ be obtained by replacing $K(x,\xi)$ with $K_x(x,\xi)$ in the limiting process which gives $\rho(x)$. If then $\rho_1(x)\neq 0$, $f''^{-}(x)$ and $K_{xx}^{-}(x,\xi)$ both exist, there is no solution of (1) unless $f(0)=f'(0)=0$. If this last condition holds, equation (1) has a unique bounded solution given by a series involving $f''^{-}(x)$, $K_{xx}^{-}(x,\xi)$, and $\rho_1(x)$. (Received March 12, 1931.)

173. Dr. Wladimir Seidel (National Research Fellow): *On analytic functions with infinitely many zeros.*

Let $f(z)$ be a regular, analytic function defined in the unit circle $|z|<1$. Consider a sequence of points $z_1, z_2, \cdots, z_n, \cdots$ lying in the interior of the unit circle and converging toward the point $z=1$. If $\lim_{n\to\infty}f(z_n)$ exists and is equal to α, we shall call α a *cluster value* of $f(z)$ in $z=1$, and we shall call the set of all cluster values of $f(z)$ in $z=1$ the *cluster set* of $f(z)$ in $z=1$. The author proves the following theorem: *Let $w=f(z)$ be a bounded analytic function: $|f(z)|<1$ in the unit circle $|z|<1$. Let $z_1, z_2, \cdots$ and $z_1', z_2', \cdots$ be two sequences of interior points of the unit circle converging toward $z=1$, such that the non-euclidean distance $D(z_n, z_n')$ is less than a constant M, independent of n: $D(z_n, z_n')<M$, $n=1, 2, \cdots$. Then $f(z)$ always has at least one of the following two properties*: (i) *If* $\lim_{n\to\infty}f(z_n)=\alpha$, *then* $\lim_{n\to\infty}f(z_n')=\alpha$; (ii) *The cluster set of $w=f(z)$ in $z=1$ contains a region R of the w-plane bounded by a closed Jordan curve and each value w, save at most one, of the region R is assumed infinitely often by $f(z)$ in the interior of $|z|<1$.* (Received March 13, 1931.)