GEODESIC COORDINATES OF ORDER r*

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1. *Introduction.* Let $\Gamma^i_{\alpha\beta}$ be a general symmetric affine connection† and $\Gamma^i_{\beta_1\beta_2\cdots\beta_r}$, $(r=3, \cdots, p)$, the sequence of the first $p-2$ generalized symmetric affine connections defined by‡

$$\Gamma^i_{\alpha\beta\gamma} = \frac{1}{3}P\left(\frac{\partial\Gamma^i_{\alpha\beta}}{\partial x^\gamma} - 2\Gamma^i_{\sigma\alpha}\Gamma^\sigma_{\beta\gamma}\right)$$

and in general by the recurrence formula

$$\Gamma^i_{\alpha\beta\gamma\cdots\delta\epsilon} = \frac{1}{N}P\left(\frac{\partial\Gamma^i_{\alpha\beta\gamma\cdots\delta}}{\partial x^\epsilon} - \Gamma^i_{\sigma\beta\gamma\cdots\delta}\Gamma^\sigma_{\alpha\epsilon} - \cdots - \Gamma^i_{\alpha\beta\gamma\cdots\sigma}\Gamma^\sigma_{\delta\epsilon}\right).$$

The law of transformation of the affine connection $\Gamma^i_{\alpha\beta}$ is, as is well known,

$$\overline{\Gamma}^\sigma_{\alpha\beta}\frac{\partial x^i}{\partial \bar{x}^\sigma} = \Gamma^i_{\lambda\mu}\frac{\partial x^\lambda}{\partial \bar{x}^\alpha}\frac{\partial x^\mu}{\partial \bar{x}^\beta} + \frac{\partial^2 x^i}{\partial \bar{x}^\alpha \partial \bar{x}^\beta},$$

while the generalized affine connections transform in accordance with the law§

$$\overline{\Gamma}^\sigma_{\alpha\cdots\beta}\frac{\partial x^i}{\partial \bar{x}^\sigma} = \frac{\partial^p x^i}{\partial \bar{x}^\alpha\cdots\partial \bar{x}^\beta} + \Gamma^i_{\lambda\cdots\mu}\frac{\partial x^\lambda}{\partial \bar{x}^\alpha}\cdots\frac{\partial x^\mu}{\partial \bar{x}^\beta} + [\quad]$$

where the [ ] denotes the sum of terms, each of which involves a component $\overline{\Gamma}^i_{\rho\cdots\sigma}$ with less than p subscripts, that vanish with $\overline{\Gamma}^i_{\rho\alpha}, \cdots, \overline{\Gamma}^i_{\rho\cdots\sigma\beta}$.

2. *Fundamental Theorems.*

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† We assume that the reader is conversant with the tensor theory as presented by O. Veblen in his *Invariants of Quadratic Differential Forms*, Cambridge Tract, 1927.

‡ O. Veblen and T. Y. Thomas, Transactions of this Society, vol. 25 (1923), p. 561.

§ T. Y. Thomas, American Journal Mathematics, vol. 50 (1928), p. 518.