A PROPERTY OF CONTINUA EQUIVALENT TO LOCAL CONNECTIVITY*

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1. Introduction. It is the purpose of this note to develop a property of continuous curves which seems to have been overlooked by the numerous writers on this subject and which, in effect, gives a new definition of this extensive class of continua. The property in question is given by the following definition.

A continuum (or space) M is said to be divisible if, for every pair of sub-continua A and B without common points, there is a decomposition of M into two continua P and Q such that $P \cdot B = Q \cdot A = 0$.

It will be shown that for a compact, metric, and connected space the concepts of divisibility and local connectivity at every point are equivalent. It will also be shown that this is true for any continuum, bounded or unbounded, located in a euclidean space.

As a preliminary we note that we cannot replace the word "sub-continua" in the above definition by the word "points." To see the truth of this statement consider the plane continuum M consisting of a segment ab of length 1 and an infinite set of arcs of radii n $(n=2, 3, \cdots)$, each subtended by ab and of length less than a semi-circumference. It is readily seen that this continuum is not divisible in the sense of the above definition, but that for any two *points* A and B we can decompose M into two continua P and Q such that $P \cdot B = Q \cdot A = 0$.

2. THEOREM. Let M be a metric, separable, connected space which is locally connected at each point. Then M is divisible.

PROOF. Let A and B be any proper sub-continua of M and $A \cdot B = 0$. Let x be any point of A, let $\epsilon > 0$ be less than one-third the distance between x and B, and let the symbol $V_{\epsilon}(x)$ denote the set of points of M whose distance from x is less than

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