ON THE LOCATION OF ROOTS OF POLYNOMIALS AND ENTIRE FUNCTIONS*

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On account of its importance, the problem of the location of the roots of a polynomial occupies the central position in the theory of equations. For the most part the attention of mathematicians has been confined to the real roots of real polynomials—i.e., polynomials with real coefficients. So general has been the assumption of the reality of the polynomial that it has often, unfortunately, not been stated in theorems in which it is vital; for example, when it is said that imaginary roots enter in conjugate pairs. Correspondingly, the tools for the investigations were ordinary algebra and the calculus. When, however, one's thought is extended to the location of imaginary roots and to unreal polynomials, the border line is crossed, as the treatment involves almost inevitably considerations and methods belonging to the theory of analytic functions. It is in the overlapping region belonging alike to the theory of equations and that of analytic functions that we will stray today.

After the notable advances of Budan, Fourier, Cauchy, Laguerre and others, investigation of the root-location of polynomials and entire functions was largely neglected for other work. With the new century has come a new activity. It is my pleasure to bring before you today some of the important advances made and some theorems discovered since 1900. This will, I hope, be the more profitable because the new results lie scattered and untied through the periodical literature and have not yet become a part of current texts.† Some of them, though little known today, are destined to become classic.

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† Except Bieberbach's noteworthy *Vorlesungen über Algebra*, 1928.