GROUPS WHICH ADMIT AUTOMORPHISMS IN WHICH EXACTLY THREE-FOURTHS OF THE OPERATORS CORRESPOND TO THEIR INVERSES

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It is well known that a necessary and sufficient condition that a given group G be abelian is that each of the operators of G correspond to its inverse in one of the possible automorphisms of G, and that in an automorphism of an abelian group all the operators must correspond to their inverses if more than half of them satisfy this condition. Moreover, it is known that in a non-abelian group at most three-fourths of the operators correspond to their inverses in a possible automorphism. The object of the present article is to determine some fundamental properties of the category of groups which separately admit an automorphism in which exactly three-fourths of the operators correspond to their inverses. Such an automorphism will be called a *three-fourths automorphism*.

Suppose that G is a group for which a three-fourths automorphism has been established, and that s_1 and s_2 are any two of its operators which are both non-commutative and correspond to their inverses in this automorphism, while s_α is any operator of G which corresponds to its inverse in the same automorphism. A necessary and sufficient condition that s_1s_α correspond to its inverse in this automorphism is that s_1 and s_α be commutative. Since s_1s_α must correspond to its inverse for at least two-thirds of the possible values of s_α it results that s_1 is commutative with exactly one-half of the operators of G, and that these operators constitute an abelian subgroup of G since all of its operators correspond to their inverses in an automorphism. Since an abelian subgroup of index 2 must also correspond