LANDAU ON NUMBER THEORY

Vorlesungen über Zahlentheorie. By Edmund Landau. Volume I: *Aus der elementaren und additiven Zahlentheorie.* xii+360 pp. Volume II: *Aus der analytischen und geometrischen Zahlentheorie.* vii+308 pp. Volume III: *Aus der algebraischen Zahlentheorie und über die Fermatsche Vermutung.* vii+341 pp. Leipzig, S. Hirzel, 1927.

These three excellently printed and arranged volumes form an addition of the highest importance to the literature of the theory of numbers. With them, the reader familiar with the basic elements of the theory of functions of a real and complex variable, can follow many of the astonishing recent advances in this fascinating field. His interest is enlisted at once and sustained by the accuracy, skill, and enthusiasm with which Landau marshals the analytic facts and simplifies as far as possible the inevitable mass of details.

Part I of Volume I gives a treatment of the elements of the theory of numbers up to the theory of quadratic residues and of the Pellian equation. This occupies a little over 50 pages. In Part II is first deduced the elementary inequality

$$a_1\xi/\log \xi < \pi(\xi) < a_2\xi/\log \xi$$

for the number of primes $\pi(\xi)$ not greater than ξ, after which follows the proof of the theorem of Brun (1919) that, if the number of "twin primes" p, $p+2$ is infinite, these are at least so infrequently distributed that the series of reciprocals

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{11}+\frac{1}{13}+\cdots$$

converges, in contradistinction with the series of the reciprocals of all primes, which diverges. The reader is then introduced in the same part II to characters $\chi(a)$ mod k, and the associated Dirichlet series, by means of which is established Dirichlet's classical theorem that every arithmetical progression $l+ks(s=1, 2, \cdots)$ contains infinitely many primes if l and k are relatively prime.

All of this material in Parts I and II, which acquaints the reader with some of the most interesting ideas in number theory, occupies slightly less than 100 pages.

In Part III are obtained the classical theorems concerning the representation of an integer as the sum of two, three, or four squares, and in Part IV are developed the classical results concerning the class number of binary quadratic forms.

Part V presents that part of the recent extraordinary series of researches, *Some Problems of "Partitio Numerorum,"* Parts I–V (1920–1925), by Hardy and Littlewood, which lies in the direction of Goldbach's conjecture that every even integer >2 can be expressed as the sum of two odd