ON A CERTAIN METRIC ASPECT OF PLANE PROJECTIVE TRANSFORMATIONS*

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1. *Introduction and Results.* This paper contains the study of a special type of metric relations between projective planes. To bring out clearly the point of view from which the planes are considered, we first briefly summarize the principal known facts about the subject in general, remarking that by "metric" relations we refer merely to properties based on the ordinary euclidean measure of a straight line segment and an angle.

Let π and π' be two projectively related planes.† At a general point of $\pi(\pi')$ there are ∞^1 angles and on a general line ∞^1 segments which preserve their size under the transformation. These angles and these segments form an involution at the point and on the line.

There exist, however, in π and π' two distinguished pairs of corresponding points, the focal points, and two distinguished pairs of corresponding lines, the equi-segmental axes, such that all corresponding angles at these points and all corresponding segments on these lines are congruent. In either plane, these points and lines lie symmetrically with regard to the vanishing line. If the distance of the focal points from the vanishing line is c in π and c' in π', then the distance of the equi-segmental axes from this line is c' in π and c in π'. The quantities c and c' are called the parameters of the transformation, and the line joining the focal points in either plane is called the focal axis.

Thus in two projective planes, the metric properties of

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† For the following see H. J. S. Smith, *Memoir on the focal properties of homographic figures*, Collected Works, vol. 1, p. 545–602. We assume with Smith throughout this paper that the projective transformation considered is a general one, and we exclude the affine case.