ON THE ATTRACTION OF SPHERES IN ELLIPTIC SPACE*

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1. *Introduction.* C. Neumann, Killing, and Liebmann have treated the motion of a material particle about a center of attraction in elliptic (hyperbolic) space. The question arises do these results hold when the center of attraction is replaced by a spherical mass.

Let the sphere be placed at the origin of coordinates O, let the polar coordinates of an element of volume at P of the sphere be ρ, ϕ, θ, where ϕ, θ are latitude and longitude. The element of volume is then

$$dv = \sin^2 \rho \cos \phi d\theta d\rho d\phi,$$

where for simplicity we take the space constant $R=1$. We will suppose the elementary mass A attracted by the sphere is on the z axis. Let $OA = \alpha$, $AP = \epsilon$, in elliptic measure. The force of attraction we will take to be

$$F = \frac{cdv}{\sin^2 \epsilon}, \qquad c \text{ a constant.}$$

If ψ is the angle AP makes with the z axis, the work done by the force F for a small displacement of A of extent $\delta\alpha$ along the z axis is

$$\delta W = F \cos \psi dv \cdot \delta\alpha.$$

It will be convenient to set

$$a = \sin \alpha, \quad r = \sin \rho, \quad e = \sin \epsilon, \quad p = \sin \phi,$$
$$a' = \cos \alpha, \quad r' = \cos \rho, \quad e' = \cos \epsilon, \quad p' = \cos \phi.$$

We have then

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