THE EQUATION OF THE V_{n-1}^{n-1} IN S_n*

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Segre, dealing with synthetic and enumerative geometry, has twice mentioned† that the manifold formed by all the lines of S_n which meet n given S_{n-2}'s is a V_{n-1}^{n-1}. In a recent article, Wong‡ mentions that the equation of the general V_{n-1}^{n-1} is unknown. It is the purpose of this paper to exhibit the equation.

The method is the following: through a general point of one of a set of n given S_{n-2}'s is passed a line which is required to meet each of the remaining $(n-1)$ S_{n-2}'s. The eliminant of the system of equations thus set up is the desired equation, as will shortly appear.

Let the equations of the n given S_{n-2}'s be

$$x_i = 0 = \sum_{j=0}^{j=n} a_{ij}x_j, \qquad (i = 1, 2, \cdots, n), \qquad (a_{ii} = 0).$$

Berzolari§ has shown that the above display does not particularize the set of S_{n-2}'s, but is a mere question of a suitable choice of the reference system. We shall need the Grassmann-Plücker coordinates of the S_{n-2}'s, that is, the two-rowed determinants from the matrices of the coefficients in their equations. In each set of coordinates we here find $n(n-1)/2$ elements are zero; the remaining n are some of the numbers a_{ij}, prefaced with a proper sign.

Let y be the coordinates of a point on one of the S_{n-2}'s, say the first one. We then have that

$$\text{(A)} \qquad y_1 = 0, \qquad \sum_{j=0}^{j=n} a_{1j}y_j = 0.$$

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† Segre, *Mehrdimensionale Räume*, Encyklopädie, vol. III C7, pp. 815 and 832.

‡ Wong, B. C., this Bulletin, vol. 34 (1928), pp. 553–554.

§ Berzolari, Rendiconti del Circolo Matematico di Palermo, vol. 29 (1905), p. 229.