ON THE MAPPING OF THE QUADRUPLES OF THE INVOLUTORIAL G_4 IN A PLANE UPON A STEINER SURFACE*

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1. *Introduction.* Castelnuovo has definitely shown† that every plane involution may be mapped uniformly upon a rational surface. As may be expected, and as the author has shown in case of the involution of sextuples,‡ from such a mapping process arise interesting properties of certain configurations and curves which reflect geometric properties of the corresponding surface, and conversely. In this paper the involution induced by the group

$$G_4 \equiv \begin{pmatrix} \pm x_1, & \pm x_2, & \pm x_3 \\ x_1, & x_2, & x_3 \end{pmatrix}$$

is investigated from this standpoint.

In what follows I shall denote the involutorial quadruple in the plane (x) merely by G_4. To construct this, let $A_1(1, 0, 0)$, $A_2(0, 1, 0)$, $A_3(0, 0, 1)$ be the coordinate triangle, and $B(x_1, x_2, x_3)$ a generic point. Join B to A_1, A_2, A_3 and construct the fourth harmonic lines to BA_1, BA_2, BA_3 with respect to the pairs of sides A_1A_2, A_1A_3; A_2A_3, A_2A_1; A_3A_1, A_3A_2. These three lines intersect in the triangle $B_1(-x_1, x_2, x_3)$, $B_2(x_1, -x_2, x_3)$, $B_3(x_1, x_2, -x_3)$, which together with $B(x_1, x_2, x_3)$ form the G_4. If we construct for each B_i the symmetric G_6 and denote it by (B_i), we obtain the configuration of the octahedral group

$$G_{24} \equiv \begin{pmatrix} \pm x_i, & \pm x_k, & \pm x_l \\ x_1, & x_2, & x_3 \end{pmatrix},$$

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† *Sulla razionalità delle involuzioni piane,* Mathematische Annalen, vol. 44 (1894), pp. 125–155.

‡ *On the mapping of the sextuples of the symmetric substitution group G_6 in a plane upon a quadric,* this Bulletin, vol. 33 (1927), pp. 745–750.