ON THE POTENTIAL OF CERTAIN SURFACE DISTRIBUTIONS

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The surface distributions that are commonly studied in potential theory are those known as *single* and *double layers*, or as surface distributions of *poles* and *doublets*. A. Wangerin† first investigated potentials of what he called triple layers (dreifach belegte Flächen), that is, potentials of the form

$$v(P) = \int \mu(\Pi) \frac{\partial^2[1/r(P,\Pi)]}{\partial n^2} dS. \tag{1}$$

Here $r(P, \Pi)$ is the distance between a point Π of the surface S and a point P in space, dS is the element of area of S at Π, $\mu(\pi)$ is a given function along S (the density function), and $\partial/\partial n$ indicates differentiation along the normal toward a specified side of S that might be called its positive side.‡

He showed that both v and its first derivatives are discontinuous at S. M. Freund§ studied more complex distributions over a spherical surface of the form

$$v(P) = \int \mu(\Pi) \frac{\partial^k[1/r(P,\Pi)]}{\partial n^k} dS. \tag{2}$$

It is the object of this paper to point out that by means of certain integrations by parts these integrals may be replaced, in case the surface S is closed, by other surface integrals

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† A. Wangerin, *Über das Potential dreifach belegter Flächen*, Jahresbericht der Vereinigung, vol. 29 (1920), p. 174.

‡ The considerations that lead to the name triple layers are analogous to the ones that suggest the double layer nomenclature: if the derivative is replaced by the limit of a proper quotient, v is seen to be the limit of the potential of three single layers spread over three nearby surfaces.

§ M. Freund, *Über das Potential mehrfach belegter Flächen*, Dissertation, Universität Halle (1922).