THE DERIVATION OF ALGEBRAIC INVARIANTS BY TENSOR ALGEBRA*

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1. *Review of Elements of Tensor Algebra.* The few simple laws of tensor algebra offer a basis for a very natural approach to the theory of algebraic invariants. In the study of algebraic invariants a ground form is chosen as, for example, the bilinear n-ary form $a_{\rho\sigma}u^\rho v^\sigma$‡ and the quantities u^r and v^s are transformed linearly by cogredient transformations such as

$$\bar{u}^r = u^\rho q_\rho^{\ r}. \tag{1}$$

If the equivalent transformed form be now written $\bar{a}_{\rho\sigma}\bar{u}^\rho\bar{v}^\sigma$, the transformation equations for the a's are

$$\bar{a}_{rs} = a_{\rho\sigma}p_r^{\ \rho}p_s^{\ \sigma}, \tag{2}$$

where the p's satisfy the equations

$$p_s^{\ r}q_\rho^{\ r} = p_\rho^{\ r}q_s^{\ \rho} = \delta_s^r, \qquad (= 0,\ r \neq s\ ;\ = 1,\ r = s), \tag{3}$$

or what is equivalent, $q_s^{\ r}$ is equal to the cofactor of $p_s^{\ r}$ in the determinant $|p_s^{\ r}|$, divided by this determinant. In tensor algebra the two equations of transformation (1) and (2) illustrate two types of tensors, that is, sets of ordered functions which are transformed in this linear manner. The set of quantities u^ρ is called a *contravariant tensor of rank* 1 since the transformed quantities are each expressed linearly in the q's. The set of quantities a_{rs} with two lower indices

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‡ Repeated Greek letters are summed from 1 to n. Such a form is ordinarily written $a_{\rho\sigma}x^\rho y^\sigma$. In the theory of surfaces, with the constant values at a point, of a given tensor such as a_{rs} may be associated arbitrary vectors, in this case two, u^r and v^s. Thus when a general transformation of coordinates is made the vectors are transformed linearly, so the theory of algebraic invariants is applicable at the point.