ON THE BEHAVIOR OF INTEGRAL FUNCTIONS IN DISTANT PORTIONS OF THE PLANE*

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1. *Introduction.* In a series of remarkable papers extending over the years 1904–08 Mr. E. W. Barnes† has determined the asymptotic behavior in the neighborhood of the point at infinity of various types of analytic functions. The types considered are distinguished from each other by the forms of their Maclaurin developments, these being regarded as given and forming the starting point of the investigation in each case.‡ In point of method, Barnes makes extensive use of contour integrations and the calculus of residues. In point of results, his emphasis falls rather upon the detailed study of each individual function-type considered than upon the attainment of one or more general theorems which may be regarded as central in character to the subject as a whole. In my book on divergent series (see footnote below) I have already indicated on page 60 a general theorem through which the asymptotic character of a wide variety of special functions, including some of those considered by Barnes, may be readily determined§ and I desire in the present paper to indicate another such theorem, this being of a nature supplementary to the former one. It may be applied

* Presented to the Society, September 8, 1927.

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‡ A list of the papers referred to may be found in my book entitled *Studies on Divergent Series and Summability* (Michigan Science Series, vol. II, 1916) page 184. The various function-types considered by Barnes are summarized in his paper entitled *The asymptotic expansion of integral functions defined by Taylor's series* appearing in the Philosophical Transactions of the Royal Society of London, vol. 206 (1906), pp. 249–297.

§ I take this occasion to note that the relation $w=0, 1, 2, 3, \cdots$, occurring in line 6 of the theorem should be corrected to read $w=0, \pm 1, \pm 2, \pm 3, \cdots$.