

ON THE LAW OF EXCLUDED MIDDLE

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1. *Introduction.* The purpose of this paper is to discuss the possibility of a system of logic in which the law of excluded middle is not assumed, and also to point out what seem to be errors in a recent paper† in which the conclusion is reached that such a system of logic is self-contradictory.

The law of excluded middle is the logical principle in accordance with which every proposition is either true or false. This principle is used, in particular, whenever a proof is made by the method of *reductio ad absurdum*. And it is this principle, also, which enables us to say that the denial of the denial of a proposition is equivalent to the assertion of the proposition.

The simplest alternative to the inclusion of the law of excluded middle among the principles of logic is its bare omission without assertion of any contrary principle. The effect of such an omission is, of course, to reduce the number of theorems which can be proved and also to render of interest certain theorems otherwise vacuous. We cannot derive theorems which contradict theorems obtained with the aid of the law of excluded middle unless we make some assertion of a principle which contradicts the law of excluded middle.

It is not possible, as an alternative to the law of excluded middle, to assert that some proposition is neither true nor false, because by so doing not only the law of excluded middle would be denied but also the law of contradiction. In fact, to assert that a proposition p is not true and is also not false is to assert at once *not- p* and *not-(not- p)* and consequently to assert that *not- p* is both true and false.

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† M. Barzin and A. Errera, *Sur la logique de M. Brouwer*, Académie Royale de Belgique, Bulletins de la Classe des Sciences, (5), vol. 13 (1927), pp. 56-71.