

## SOME PROPERTIES OF CONTINUOUS CURVES\*

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The points  $A$  and  $B$  of a continuum  $M$  are said to be separated in  $M$  by a point  $X$  of  $M$  if  $M - X$  is the sum of two mutually separated sets  $S_1$  and  $S_2$  containing  $A$  and  $B$  respectively. The point  $P$  of a continuum  $M$  is a cut point of  $M$  if and only if the set of points  $M - P$  is not connected, i.e., is the sum of two mutually separated point sets.

A continuous curve  $M$  will be said to be *cyclicly connected* provided that every two points of  $M$  lie together on some simple closed curve which is contained in  $M$ . In this paper use will be made of the following fundamental theorem.

**THEOREM A.** *In order that the continuous curve  $M$  should be cyclicly connected it is necessary and sufficient that  $M$  should have no cut point.*

A proof for Theorem A will be found in my paper *Cyclicly connected continuous curves*, which will appear soon.

**THEOREM I.** *If  $A$  and  $B$  are any two points of a continuous curve  $M$  and if  $K$  denotes the set of all those points of  $M$  which separate  $A$  from  $B$  in  $M$ , then  $K + A + B$  is a closed set of points.*

**PROOF.** The curve  $M$  contains a simple continuous arc  $t$  from  $A$  to  $B$ . Clearly  $K$  must be a subset of  $t$ . Let  $P$  be any point of  $t - (K + A + B)$ . Since  $P$  does not belong to  $K$ ,  $A$  and  $B$  must both belong to some connected subset<sup>†</sup> of  $M - P$ , and by a theorem of R. L. Moore's<sup>‡</sup> it follows that  $M - P$  contains an arc  $t'$  from  $A$  to  $B$ . On the arcs  $PA$

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† See an abstract of a paper by R. L. Wilder, *A characterization of continuous curves by a property of their open subsets*, this Bulletin, vol. 32 (1926), pp. 217-218.

‡ *Concerning continuous curves in the plane*, Mathematische Zeitschrift, vol. 15 (1922), p. 255.