SINGULARITIES OF THE HESSIAN*

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1. *Introduction*. It has been proved† that when a curve f has no point singularities its Hessian H has no point singularities. Then point singularities occur on H when and only when f has point singularities. Moreover, singular points of H can occur only at those points which are singular points of f.

The number of intersections of f and H at any singularity of f is

$$6\delta_1 + 8\kappa_1 + \iota_1$$

where δ_1, κ_1, ι_1 are the numbers of nodes, cusps, and inflections respectively contained in the singularity of f. A given singularity of f needs but to be resolved and the number of intersections of f and H are thus found without reference to H. In order for this number of intersections to occur, there must be a singularity of H at this point, but except for cusps and simple multiple points with distinct tangents these singularities of H have not been investigated.

The purpose of this paper is to explain geometrically how the intersections of f and H at a given singularity of f occur. The principal problem involved is to find the singularity of H corresponding to a given singularity of f.

2. *Simple Multiple Points*. It has long been known that at a simple r-fold point of f with r distinct tangents, H has a $(3r-4)$-fold point, r of whose tangents coincide, one each, with r tangents of the r-fold point on f; also that at a cusp of f, H has a triple point two of whose tangents coincide with the cuspidal tangent.

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† A. B. Basset, *On the Hessian, the Steinerian, and the Cayleyan*, Quarterly Journal, vol. 47 (1916), p. 227.