ON THE INTEGRO-DIFFERENTIAL EQUATION OF THE BÔCHER TYPE IN THREE-SPACE

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1. *Introduction.* Bôcher has shown* that if a function $f(x, y)$ is continuous and has continuous first partial derivatives in a region R and satisfies the condition

$$\int_C \frac{\partial f}{\partial n} ds = 0$$

for every circle C lying entirely in R, then $f(x, y)$ is harmonic at each interior point of R. Bôcher treats only functions in two variables and by a method which cannot be directly extended to three-space.

It is the purpose of the present note to show, by a simple modification of the second part of Bôcher's argument, that this result may at once be extended to three-space, and also to investigate the nature of the function f if Bôcher's condition of continuity is somewhat weakened. We shall treat explicitly functions in three variables only, but it will easily be seen that with a slight modification the statements of Theorem II are applicable to two-space as well.

THEOREM I. *If a function $f(x, y, z)$ is continuous, and has continuous first partial derivatives in a connected finite region R, and is such that the surface integral $\int_S (\partial f/\partial n)ds$ vanishes when taken over every sphere S lying in R, then at each interior point of R, f is harmonic; that is, it satisfies Laplace's equation*

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

at each interior point of R.

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* PROCEEDINGS OF THE AMERICAN ACADEMY, vol. 41, pp. 577–583.