ON THE POLYNOMIAL OF THE BEST APPROXIMATION TO A GIVEN CONTINUOUS FUNCTION*

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1. *A Theorem on Minimizing Polynomials.* Let $f(x)$ and $p(x)$ be defined on a finite interval (a, b); $f(x)$ is bounded and integrable, $p(x)$ is integrable and not negative.

THEOREM I.† *If there exist two numbers α, β such that $a \leqq \alpha < \beta \leqq b$, and such that*

$$\int_c^d p(x)\,dx > 0$$

whenever $\alpha \leqq c < d \leqq \beta$, then there exists one and only one polynomial of degree $\leqq n$ minimizing the integral

$$I_{nk} = \int_a^b p(x)\,|f(x) - U_{nk}(x)|^k\,dx,$$

where

$$U_{nk}(x) = \sum_{i=0}^{n} u_{ik} x^i,$$

provided that $k > 1$. If $k = 1$, the proof of existence applies without change; and the approximating polynomial is unique, if $f(x)$ is continuous on (a, b), and if

$$\int_c^d p(x)\,dx > 0$$

whenever $a \leqq c < d \leqq b$.

The proof may be organized as follows.

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† Cf. G. Pólya, *Sur un algorithme ...*, COMPTES RENDUS, vol. 157 (1913), pp. 840–843; D. Jackson, *On functions of closest approximation*, TRANSACTIONS OF THIS SOCIETY, vol. 22 (1921), pp. 117–128, *Note on a class of polynomials of approximation*, ibid., vol. 22 (1921), pp. 320–326, *A generalized problem in weighted approximation*, ibid., vol. 26 (1924), pp. 133–154, *Note on the convergence of weighted trigonometric series*, this BULLETIN, vol. 29 (1923), pp. 259–263.