

Einleitung in die Mengenlehre. 2d edition. By A. Fraenkel. Berlin, Julius Springer, 1923. viii + 251 pp.

In this edition this book appears as volume 9 of the series, *Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen*. The author in writing this edition tried to produce a book which would fulfill the general requirements of that series and also the purpose of the first edition*, i. e. to give an exposition of the elements of Mengenlehre which would be clearly comprehensible to one of little or no mathematical training. The prolixity of exposition necessary in realizing the latter aim hinders the realization of the former. The elegance and, in particular, the conciseness which the initiated desires is missing. However, the author seems to have come as near as possible to fulfilling with a single book both purposes.

The first 151 pages of the new edition contain almost verbatim all of the material of the first 129 pages of the old edition and in addition the introduction of certain concepts, more abstruse than those of the earlier book, and the details of certain proofs which previously were given only in outline. The material of chapter 12, pp. 129–151, of the old edition is elaborated in 90 pages, pp. 151–241, of the 2d edition. These pages are devoted to the foundations of the subject. About half of this space is taken up with a historical sketch of the critical examination of the theory, including an exposition of the paradoxes which gave rise to this critique and a brief description of the various procedures devised to give a logically consistent theory. Here are briefly explained and contrasted intuitionism, which recently has been the subject of considerable work by Brouwer and Weyl, the method of logicizing as developed by Russell and Whitehead and J. Königs, and the axiomatic method as employed by Zermelo and Hilbert. The remaining half is devoted to a detailed exposition of the axiomatic development of Mengenlehre which Zermelo produced in the period from 1904–1908 and to a sketch of Hilbert's recent work on the question of a proof of the consistency of a system of postulates. In connection with the Zermelo axiomatic the author gives in fine print an exposition of his contribution to the clarifying of the notion "eine definite Aussage" as used by Zermelo.

The exposition of the Zermelo–Hilbert way of setting up a theory of sets is well done but the brevity with which the other methods, logicization and intuitionism, are presented makes it impossible to give an exposition of the latter methods on a par with that of the former. This is to be regretted. However, in these 90 pages we have the most readily available means of obtaining an acquaintance with the present state of the theory of the foundations of Mengenlehre

* See this BULLETIN, vol. 27 (1921), pp. 333–334.