Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik. By Adolph Kneser. 2d edition. Braunschweig, Vieweg, 1922. 8+292 pp.

Kneser's *Integralgleichungen*, which appeared first in 1911, was a pioneer in its field, and during the time which has elapsed since then, it has remained the only thing of its kind. To be sure, the book of Heywood and Fréchet emphasizes the physicial applications, and the problems of potential theory, in particular, have been methodically developed by means of integral equations. But the present work is, as far as the reviewer knows, unique in its inclusion of and emphasis on the application of the Hilbert-Schmidt theory to important one-dimensional problems.

The first edition was ably reviewed in the BULLETIN* by W. A. Hurwitz. The changes which have been made consist in about eighty pages of new or rewritten material, the remaining pages of the text having been reprinted without other change than an occasional alteration in notation. The most significant feature of the revision is the assembling and rounding out in one chapter (III) of a general theory of the symmetric kernel. The presentation follows the lines of the Schmidt theory, but it is further enhanced by the addition of paragraphs on Mercer's theorem on the convergence of the development of a continuous definite kernel in terms of its characteristic functions, and on Weyl's theorem on the diminishing effect on the positive characteristic numbers of a kernel caused by adding to the kernel a positive definite kernel.

The author finds a physical problem leading to an unsymmetric kernel in the vibrations of an elastic bar, account being taken of thermal effects. For this problem, the development question is treated by means of the theory of residues. Difficulties due to multiple characteristic numbers are not here encountered. There follow indications for a similar treatment of Sturm-Liouville problems in which the coefficients are not always necessarily real. The case in which the fundamental interval extends to infinity is then considered, and Hilb's generalization of the Fourier integral is derived and illustrated in the known cases of the Fourier integral and the corresponding integral in Bessel functions.

Another new topic arises from the treatment of a boundary value problem connected with electric cables, in which a boundary condition depends on the parameter. Here, ordinary orthogonality is replaced by a

* Vol. 19 (May, 1913), pp. 406–11. For other analyses and reviews, see *Lacour*, BULLETIN DES SCIENCES MATHÉMATIQUES, vol. 46 (1911), pp. 254–61; Korn, ARCHIV FÜR MATHEMATIK UND PHYSIK, (3), vol. 18 (1911), pp. 82–83; Plancherel, L'ENSEIGNEMENT MATHÉMATIQUE, vol. 13 (1911), pp. 428–29.