BOREL'S THEORY OF FUNCTIONS.

Méthodes et Problèmes de la Théorie des Fonctions. By Émile Borel.
   (Collection de monographies sur la théorie des fonctions.)   Paris,
   Gautier-Villars, 1922.   ix + 148 pp.

The volume under review, which Borel states in the introduction is
to be the last of this justly celebrated collection of monographs to
appear under his own name, is designed to supplement the earlier
volumes of the series by bringing together in their original form a
number of his notes and memoirs on the theory of functions.   The
twenty-seven articles included in this volume appeared in a number
of journals during the years 1895—1912.   They cover a wide range
of subject matter and indicate clearly the breadth, originality, and
versatility which have been the source of the very great influence which
Borel has exerted on the progress of analysis in the last thirty years.

The memoirs presented are grouped in four chapters: *Les domaines
et la théorie des ensembles*; *Les opérations et les développements en séries*;
La théorie de la croissance et le rôle des constants arbitraires; *Les
fonctions de variable complexe, en général, et les fonctions particulières.*

The introduction is devoted to an interesting analysis of the many
striking analogies between biology and the theory of functions, both in
content and in historical order of development.   As man learned to use
animals and plants before he had acquired a thoroughgoing knowledge
of anatomy or physiology so mathematicians discovered and utilized the
elementary functions, algebraic, circular, exponential.   In biology the
study of the structure of organisms led to the analysis of cellular life
and organic chemistry; in the theory of functions the analysis of the
properties of functions brought into existence the modern theories of
aggregates, numbers, sets of points.

In the first chapter Borel remarks that the domains and ensembles
are to functions as the tissues are to living organisms and calls
attention to the importance which this part of the theory of functions
has come to assume.   Of the seven papers included in this chapter
three are brief notes, one on the representation of discontinuous functions
as limits of continuous functions and two on the theory of measure.
The last of these outlines a procedure which may be substituted for
the method of Lebesgue in the theory of integration.   Three articles
are devoted to the analysis and classification of sets of measure null.
It is significant that the discoverer of the role in analysis of the sets
of measure null should be the pioneer in the study of their structure
and the closely related theory of monogenic non-analytic functions.