# THE REDUCTION OF SINGULARITIES OF PLANE CURVES BY BIRATIONAL TRANSFORMATION

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1. *Introduction.* In the domain of mathematics there are a number of famous theorems which have stimulated the keen interest of mathematicians over extended periods of time, and whose proofs have presented a long continued challenge to the powers of mathematical logic. To some of these, in spite of the fact that the theorems themselves remain unproved, our science is indebted for important advances.

As one illustration I may mention the well known theorem of Jordan which states that a simply closed continuous plane curve divides the plane into two and only two connected regions. This is a theorem of whose truth we are convinced at the start intuitively. But what we may for the moment call intuitive reasoning busies itself with simpler cases only, and is impatient of exceptions and refinements, unless by long study and continuous contemplation it has become a very sophisticated intuition indeed. Jordan was the first to insist that the theorem needed proof. Since his initial effort many others have attempted to give the conclusion of the theorem a substantial logical basis, and the theories of point sets and of functions of a real variable have been greatly enriched thereby. I am particularly interested to mention this theorem because it seems to me that with regard to it a satisfactory conclusion has been reached. There is doubtless still opportunity for improvements and simplifications in its proofs, but some at least of them have stood the test of examination by widely scattered experts. It is encouraging to have this evidence that not all of the mathematical questions generally recognized as most difficult are impossible to answer.

One should mention, of course, among the notable illustrations of the type of theorem which I have been discussing, Fermat's last theorem and the so-called four-color map